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# Transition form factors for radiative decays of heavy flavored vector mesons

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**Abstract:** We study the electromagnetic form factors of heavy flavoured vector mesons such as $(D^*, D_s^*, J/\psi), (B^*, B_s^*, \Upsilon)$ via one photon radiative decays $(V \to P\gamma)$ in the relativistic independent quark (RIQ) model based on a flavour independent average interaction potential in the scalar-vector harmonic form. The momentum dependent spacelike ($q^2 < 0$) form factors calculated in this model are analytically continued to the timelike region $0 \le q^2 \le (M_V - M_P)^2$. The coupling constant $g_{VP\gamma} = F_{VP}(q^2 = 0)$ for real photon case in the limit $q^2 \to 0$ is obtained in good agreement with experimental data and other model predictions.

**Keywords:** Electromagnetic form factors, coupling constant, spacelike and timelike region.

Electromagnetic form factors are regarded as important tools that encode information about shape of hadrons and give valuable insight into their internal structure in terms of constituent quarks and gluon degrees of freedom. Though one photon radiative decays of low lying heavy vector (V) to heavy pseudo-scalar (P) mesons have been investigated by several theoretical approaches such as quark model (QM) [1-5], light cone QCD sum rule [6,7], heavy quark effective theory (HQET) [8,9],cloudy bag model (CBM) [10] and light front quark model (LFQM) [11], not much attention has been paid to study their momentum dependent transition form factors. We have predicted the decay widths of several M1 transition $[V \to P\gamma$ and $P \to V\gamma]$, in the relativistic independent quark (RIQ) model, using a static approximation [12], in reasonable agreement with the available data in most decay modes except in those cases which involve a light flavoured meson (especially pion). The noticeable discrepancy in the prediction for such decay modes was due to the recoil effect arising out of large momentum transfer involved, which was not taken into consideration. This was considered in our subsequent analysis of radiative decay modes $[V \to P\gamma\ ,\quad P \to V\gamma]$ [2] by introducing momentum eigen states of participating mesons into our analysis as the appropriate wave packets reflecting momentum distribution of constituent quark and antiquark within the meson bound state. In going beyond the static calculation we found a significant improvement in the model predictions for decay modes involving light mesons especially pion in good agreement with the experimental data. In the same analysis our results for heavy vector to heavy pseudo-scalar meson one photon decay modes stand almost unaffected from the recoil effect and are also found to be in good agreement with other model predictions and experimental data. However we have not yet shown the momentum dependence of the relevant electromagnetic form factors as has been done in other models [1, 2, 3] including the light front quark model [11].

The purpose of this paper is to predict the space- and timelike transition form factors for energetically possible magnetic dipole decays of heavy flavoured mesons $(D^*, D_s^*, J/\psi)$ and $\left(B^*, B_S^*,\ \Upsilon\right)$in the famework of the relativistic independent quark (RIQ) model and compare with other model predictions and available experimental data. Since experimental data in heavy flavored sector are scanty, the model predictions, if found reliable, would not only justify the applicability of the model in this sector but pin down (RIQ) model as one of the most suitable phenomenological models of hadrons.

The transition form factor $F_{VP}(q^2)$ for radiative decay of vector mesons $V(p) \to P(p')\gamma^*(q)$ is defined as

$$\left\langle P(p') \,|\, J^{\mu}_{em} \,|V(p,h)\right\rangle = ie\varepsilon^{\mu\nu\rho\gamma}\varepsilon_{\nu}(p,h)F_{VP}(q^2) \qquad (1)$$

where, $q = (p' - p)$ is the four momentum transfer, $\varepsilon_{\nu}(p,h)$ is the polarization vector of the vector meson with four momentum *p* and helicity *h.* The expression for the form factor $F_{VP}(q^2)$ can be found by calculating the transition matrix element in the frame work of (RIQ) model. In the relativistic independent quark (RIQ) model, based on an average flavour independent confining potential in the scalar- vector harmonic form:[2,12,13] $U(r) = \frac{1}{2}(1+\gamma^0)(ar^2 + V_0), a > 0,$ where $(a, V_0)$ are potential parameters, the meson state is represented by an appropriate momentum wave packet in the form [2,13]:

$$|M(\vec{p},S_M)>=\frac{1}{\sqrt{N_M(\vec{p})}}\sum_{\lambda_1\lambda_2\in S_M}\xi^M_{q_1\bar{q}_2}(\lambda_1,\lambda_2)\int d^3\vec{p}_{q_1}d^3\vec{p}_{q_2}\delta^{(3)}(\vec{p}_{q_1}+\vec{p}_{q_2}-\vec{p})$$

$$\times G_M(\vec{p}_{q_1},\vec{p}_{q_2})\hat{b}^+_{q_1}(\vec{p}_{q_1},\lambda_1)\hat{\tilde{b}}^+_{q_2}(\vec{p}_{q_2},\lambda_2)|0> \tag{2}$$

where, $\hat{b}^+_{q_1}(\vec{p}_{q_1},\lambda_1)$ and $\hat{\tilde{b}}^+_{q_2}(\vec{p}_{q_2},\lambda_2)$ are respectively, the quark and antiquark creation operator and $\zeta^M_{q_1,\bar{q}_2}(\lambda_1,\lambda_2)$ is the SU(6)-spin flavour coefficients. $N_M(\vec{p})$ is the meson normalization factor of the wave packet defined as

$$N_M(\vec{p})=\int d^3\vec{p}_{q_1}\,|G_M(\vec{p}_{q_1},\vec{p}-\vec{p}_{q_1})|^2 \tag{3}$$

Finally, $G_M(\vec{p}_{q_1},\vec{p}-\vec{p}_{q_1})$ which, represents the effective momentum distribution function for the quark $q_1$ and the antiquark $\bar{q}_2$ is taken in the form [2, 13]

$$G_M(\vec{p}_{q_1},\vec{p}-p_{q_1})=\sqrt{\left(g_{q_1}(\vec{p}_{q_1})\tilde{g}_{q_2}(\vec{p}-\vec{p}_{q_1})\right)} \tag{4}$$

Here $g_{q_1}(\vec{p}_{q_1})$ and $\tilde{g}_{\bar{q}_2}(\vec{p}-\vec{p}_{q_1})$ refer to the momentum probability amplitude of the bound quark $q_1$ with momentum $\vec{p}_{q_1}$ and of the antiquark $\bar{q}_2$ with momentum $\vec{p}-\vec{p}_{q_1}$, respectively. Using appropriate momentum wave packets for the initial and final state meson as in Eq. (2), the transition matrix element in Eq. (1) is calculated to find the momentum dependent form factor $F_{VP}$ in the form

$$F_{VP}(q^2)=e_qI_q(m_q,m_{\bar{q}},q^2)+e_{\bar{q}}I_{\bar{q}}(m_{\bar{q}},m_q,q^2) \tag{5}$$

where, the model expression for $I_{q(\bar{q})}$ is obtained as

$$I\left(m_q,m_{\bar{q}},q^2\right)=\sqrt{\frac{1}{N(0)N(\vec{k})}}\int d\vec{p}_qG_V(\vec{p}_q,-\vec{p}_q)G_P(\vec{k}+\vec{p}_q,-\vec{p}_q)$$

$$\times\sqrt{\frac{(E_{p_q}+m_q)(E_{p_q+k}+E_{p_q})}{4E_{p_q}E_{p_q+k}(E_{p_q+k}+m_q)(E_{p_q}+E_{-p_q})}} \tag{6}$$

with, $k=\frac{(M_V^2-M_P^2)}{2M_V}$ is the kinematically allowed energy of the outgoing photon. Here the coupling constant $g_{VP\gamma}=F_{VP}\left(q^2=0\right)$. We use the transverse $(h=\pm1)$

polarization to extract the coupling constant $g_{VP\gamma}$ since longitudinal component of the vector meson does not convert into a real photon.

To evaluate the form factors in the allowed kinematic range, we take the potential parameter $(a, V_0)$, quark masses $m_q$ and corresponding binding energies $E_q$ as those already used in previous works describing a wide ranging hadronic phenomena in the light and heavy flavor sector [2,12,13] as

$$(a, V_0) \equiv (0.017166\ \text{GeV}^3, -0.1375)\ \text{GeV}$$

$$(m_u = m_d, m_s, m_c, m_d) \equiv (0.07875, 0.31575, 1.49276, 4.776590)\ \text{GeV} \qquad (7)$$

$$(E_u = E_d, E_s, E_c, E_b) \equiv (0.47125, 0.591, 1.57951, 4.76633)\ \text{GeV}$$

The meson masses appearing in the form factor expression are taken as their observed values [14].

**Table 1.** Coupling constant $g_{VP\gamma} = F_{VP}(q^2 = 0)$

| Coupling constant | Our work | [11] | [3] | [2] | [1] | Exp. [14] |
|---|---|---|---|---|---|---|
| $g_{J/\psi\eta_c\gamma}$ | 0.832 | 0.681[0.673] | - | - | 0.69 | $0.687 \pm 0.45$ |
| $g_{D^{*+}D^{+}\gamma}$ | -0.391 | -0.384 [-0.398] | -0.3 | -0.37 | -0.35 | $-0.466 \pm 0.3$ |
| $g_{D^{*0}D^{0}\gamma}$ | 2.056 | 1.783 [1.826] | 1.85 | 1.94 | 1.78 | - |
| $\left\lvert \frac{g_{D^{*0}D^{0}\gamma}}{g_{D^{*+}D^{+}\gamma}} \right\rvert$ | 5.26 | 4.64 [4.59] | 6.17 | 5.24 | 5.08 | - |
| $g_{D_s^{*+}D_s^{+}\gamma}$ | -0.181 | -0.167 [-0.161] | - | -0.17 | -0.13 | - |
| $g_{B^{*+}B^{+}\gamma}$ | 1.593 | 1.311 [1.313] | 1.4 | 1.50 | 1.37 | - |
| $g_{B^{*0}B^{0}\gamma}$ | -0.891 | -0.749 [0.75] | -0.8 | -0.85 | -0.78 | - |
| $\left\lvert \frac{g_{B^{*0}B^{0}\gamma}}{g_{B^{*+}B^{+}\gamma}} \right\rvert$ | 0.58 | 0.57 [0.57] | 0.57 | 0.57 | - | - |
| $g_{B_s^{*0}B_s^{0}\gamma}$ | -0.657 | -0.553 [-0.536] | - | -0.62 | -0.55 | - |
| $g_{\Upsilon\eta_b\gamma}$ | -0.138 | -0.124 [-0.119] | - | - | -0.13 | - |

We present in Table-1 our results of the coupling constant $g_{VP\gamma}$ in the limit of $q^2 = 0$, for radiative $V \to P\gamma$ decays together with other model predictions and the experimental data. The experimental values for coupling constant $(g_{J/\psi\eta_c\gamma})_{\exp} = 0.687 \pm 0.45$ for $J/\psi \to \eta_c\gamma$ and $(g_{D^{*+}D^{+}\gamma})_{\exp} = -(0.466 \pm 0.3)$ for $D^{*\pm} \to D^{\pm}\gamma$ process are extracted from the branching ratios Br $(J/\psi \to \eta_c\gamma)_{\exp}$ = $(1.7 \pm 0.4)\%$ and Br $(D^{*\pm} \to D^{\pm}\gamma)_{\exp} = (1.6 \pm 0.4)\%$) together with the full widths of $\Gamma_{total}(J/\psi) = (92.9 \pm 2.8)$ keV and $\Gamma_{total}(D^{*+}) = 83.4 \pm 1.8$ keV [14]. The opposite sign of coupling constants for $D^{*+}$ and $D_s^{*+}$ decays compared to the charmonium $J/\psi$ decay indicates that charmed quark contribution is largely destructive in the radiative decays of $D^{*+}$ and $D_s^{*+}$ mesons.

Similarly, we see that the bottomed quark contribution is largely destructive in the radiative decay of $B^{*+}$ meson. Our predicted coupling constant $g_{J/\psi\eta_c\gamma} = 0.832$ and $g_{D^{*+}D^{+}\gamma} = -0.391$ are well within the experimental error limit and those for $D^{*0} \to D^{0}\gamma$ and $D_s^{*+} \to D_s^{+}\gamma$ decays are comparable with other model predictions [1, 2, 3, 11]. Our results of coupling constant ratio $|g_{D^{*0}D^{0}\gamma} / g_{D^{*+}D^{+}\gamma}| = 5.25$ not only compares well with the model predictions [1, 2, 3, 11] as shown in Table1 but also falls within the limits of theoretical predictions of $6.32 \pm 2.97$ [6] from QCD sum rules and $5.54 \pm 3.0$ [9] from heavy quark effective theory [HQET] and $4.49 \pm 0.96$ [15] from broken SU(4) symmetry by M1 transition. Incidentally our predicted coupling constants $g_{B^{*+}B^{+}\gamma}, g_{B^{*0}B^{0}\gamma}, g_{B_s^{*0}B_s^{0}\gamma}$ and $g_{\Upsilon\eta_b\gamma}$ are also found comparable with those of [1, 2, 3, 11]. Our results of coupling constant ratio $|g_{B^{*0}B^{0}\gamma} / g_{B^{*+}B^{+}\gamma}| = 0.56$ in the bottom flavor sector is almost same as those of [1, 2, 3, 11] and fall within the limits of theoretical prediction of $0.64 \pm 0.51$ [6] and $0.49 \pm 0.38$ [7] from QCD sum rules, and $0.59 \pm 0.48$ [9] from HQET.

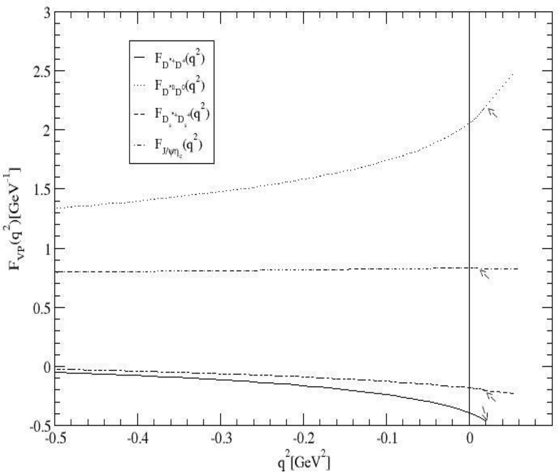


**Fig. 1:** The $q^2$ dependence of form factors $F_{VP}(q^2)$ for charmed meson radiative decay

We show in Fig. 1 our results of transition form factor $F_{VP}(q^2)$ for charmed vector meson radiative $V \to P\gamma$ decays. The solid, dotted, dashed and dot-dashed lines represent the form factors for $D^{*\pm} \to D^{\pm}\gamma^*$, $D^{*0} \to D^0\gamma^*$, $D_s^{*\pm} \to D^{\pm}\gamma^*$ and $J/\psi \to \eta_c\gamma^*$, respectively. The arrows in the figure represent the zero recoil points, where $q^2 = q^2_{\max} = (M_V - M_P)^2$ of the final state pseudo-scalar meson. We have performed the analytical continuation of transition form factors from spacelike $(q^2 < 0)$ region to the physical timelike region $(0 \le q^2 \le q^2_{\max})$. The coupling constant $g_{VP\gamma}$ at $q^2 = 0$ corresponds to a final state pseudo-scalar meson recoiling with maximum three momentum

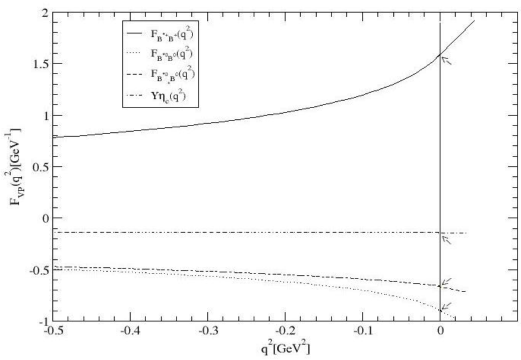


**Fig. 2:** The $q^2$ dependence of form factors $F_{VP}(q^2)$ for bottomed meson radiative decay.

$\left|\vec{k}\right| = (M_V^2 - M_P^2)/2M_V$ in the rest frame of vector meson.

Fig 2 depicts our results of transition form factors for the bottomed vector meson radiative $V \to P\gamma$ decays, where the solid, dotted, dashed and dot-dashed lines represent the form factors for $B^{*+} \to B^{+}\gamma^{*}$, $B^{*0} \to B^{0}\gamma^{*}$, $B_s^{*0} \to B_s^{0}\gamma^{*}$ and $\Upsilon \to \eta_b\gamma^{*}$, respectively. Due to small kinematic region $0 \le q^2 \le q^2_{\max}$ for the bottomed and bottomonium meson decays the recoil effects of final state meson are quite negligible, i.e. $F_{VP\gamma}(q^2_{\max})/g_{VP\gamma} \approx 1$. Likewise we find $F_{J/\psi\eta_c\gamma}(q^2_{\max})/g_{J/\psi\eta_c\gamma} \approx F_{D_s^{*\pm}D^{\pm}\gamma}(q^2_{\max})/g_{D_s^{*\pm}D_s^{\pm}\gamma} \approx 1$ for $J/\psi \to \eta_c\gamma$ and $D_s^{*\pm} \to D_s^{\pm}\gamma$ decays, respectively. On the other hand, we obtain $F_{D^{*\pm}D^{\pm}\gamma}(q^2_{\max})/g_{D^{*\pm}D^{\pm}\gamma} = 1.18$ and $F_{D^{*0}D^{0}\gamma}(q^2_{\max})/g_{D^{*0}D^{0}\gamma} = 1.06$ for $D^{*\pm} \to D^{\pm}\gamma^{*}$ and $D^{*0} \to D^{0}\gamma^{*}$ decays, respectively in good comparison with the predicted values of 1.1 and 1.04 [11] from light front quark model analysis. This shows that the recoil effect i.e. the difference between the predicted transition form factor at zero recoil point (i.e. at $q^2 \to q^2_{max}$) and those at maximum recoil point (i.e. at $q^2 \to 0$), may not be negligible in these decay modes especially in $D^{*+} \to D^{*}\gamma$ decay. Our predictions: $F_{D_s^{*+}D_s^{+}}(q^2)/F_{D^{*+}D^{+}}(q^2) \to 1$ and $F_{B_s^{*0}B_s}(q^2)/F_{B^{*0}B^{0}}(q^2) \to 1$ in the intermediate and deep spacelike ($q^2<0$) region as depicted in Fig-1 and Fig-2 indicate that the light quark current contribution in these cases comes out negligible vindicating restoration of SU(3) flavour symmetry between charmed and charmed -strange mesons as well as bottomed and bottomed-strange mesons.

We also estimate the electromagnetic radii of the form factors from the relation $\left\langle r_{vp}^2 \right\rangle = -6\frac{d}{dq^2}F_{VP}(q^2)\big|q^2 \to 0$ in the limit as $q^2 \to 0$. Our results in Table-2 show smaller values of electromagnetic radii for those decay modes involving comparatively heavier quarks as expected.

**Table-2.** Electromagnetic form factors $\langle r_{VP}^2 \rangle$ estimated in RIQ model

| Charge Radius $\langle r_{vp}^2 \rangle_{charm}$ | Our result | Charge Radius $\langle r_{vp}^2 \rangle_{bottom}$ | Our result |
|---|---|---|---|
| $\langle r_{J/\psi\eta_c}^2 \rangle$ | 0.0195 | $\langle r_{B^{*+}B^+}^2 \rangle$ | 0.0319 |
| $\langle r_{D^{*+}D^+}^2 \rangle$ | -0.0314 | $\langle r_{B^{*0}B^0}^2 \rangle$ | -0.0169 |
| $\langle r_{D^{*0}D^0}^2 \rangle$ | 0.1622 | $\langle r_{B_s^{*0}B_s^0}^2 \rangle$ | -0.0091 |
| $\langle r_{D_s^{*+}D_s^+}^2 \rangle$ | -0.0111 | $\langle r_{\Upsilon\eta_b}^2 \rangle$ | -0.0003 |

We investigated the magnetic dipole $V \to P\gamma$ decays of heavy flavoured mesons and predicted the momentum dependent electromagnetic form factors in the spacelike and timelike region in the framework of the relativistic independent quark (RIQ) model. We obtained the timelike form factor by analytical continuation of spacelike ($q^2$<0) form factor to the physical timelike ($0 \leq q^2 < q_{max}^2$)region. The coupling constants $g_{VP\gamma}$ for different decay modes are determined in the limit as $q^2 \to 0$ and our predicted values are in good agreement with other model predictions. The form factor $F_{D^{*+}D^+}(q^2)$ and $F_{D^{*0}D^0}(q^2)$ seem to have non-negligible recoil effect about 18% and 6%, respectively between zero and maximum recoil points i.e. $F_{D^{*+}D^+\gamma}(q_{\max}^2)/g_{D^{*+}D^+\gamma} \simeq 1.18$ and $F_{D^{*0}D^0\gamma}(q_{\max}^2)/g_{D^{*0}D^0\gamma} \simeq 1.06$. The corresponding ratios in the rediative decays of comparatively heavier vector mesons such as $J/\psi, D_s^{*+}, B^{*+}, B^{*0}, B_S^{*0}, \Upsilon$ are found to be unit; indicating negligible recoil effect on our results in this sector. Our predicted values of electromagnetic radii of the form factors for heavy flavoured vector meson radiative decays are obtained in the same order as expected. The prediction of coupling constant $g_{\Upsilon\eta_b\gamma}$ and hence the decay width $\Gamma(\Upsilon \to \eta_b\gamma)$ is of special interest as it would help determine the experimentally unmeasured mass of $\eta_b$ meson since the decay width $\Gamma(\Upsilon \to \eta_b\gamma)$ is very sensitive to the mass difference $\Delta m = \left(M_\gamma - M_{\eta_b}\right)$ between two bottomed mesons and is proportional to $\left(\Delta m\right)^3$. This will be explored in our future communication.

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